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Development and Application of Mathematical Model of Reactive Power Loss Index As A Measurement Of Voltage Stability in Nigerian Transmission System

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ABSTRACT

The Nigerian power system faces growing challenges in maintaining voltage stability due to increased demand, insufficient reactive power reserves, and the complexity of modern grid operations. This study presents the development and application of a mathematical model known as the Reactive Power Loss Index (RPLI) for identifying critical (weak) buses suitable for reactive power compensation in the Nigerian 52-bus transmission network. The RPLI model, derived using the bus admittance matrix method, serves as a voltage stability indicator by quantifying reactive power loss distribution across load buses. The model was implemented in MATLAB R2021a, and the voltage stability of the network was evaluated under contingency conditions. The performance of RPLI was compared with two existing methods: Relative Electrical Distance (RED) and Fast Voltage Stability Index (FVSI), based on voltage magnitude, maximum loadability, reactive power loss, and transmission line charges. Results showed that RPLI accurately identified vulnerable buses and achieved a reduction in reactive power loss by 6.78% and active power loss by 0.76% during contingencies compared to base contingency conditions. These outcomes validate the RPLI as an effective and computationally efficient tool for optimal placement of Static VAR Compensators (SVCs) and enhancing voltage stability in power systems.

INTRODUCTION

In recent years, the operation of electric power systems has become increasingly challenging due to the rapid rise in electricity demand, limited expansion of transmission infrastructure, and the growing complexity of grid operations. This is particularly evident in Nigeria, where economic development has heightened energy needs, yet the national grid continues to suffer from persistent voltage instability, inadequate reactive power support, and frequent power outages. These issues have had a detrimental impact on industrial activities, small businesses, and investor confidence in the country's energy sector (Adebayo *et al.*, 2012).

Voltage instability, which may lead to voltage collapse and system blackouts, is a major concern in heavily loaded power systems (Adebayo and Aborisade 2018, Adebayo and Sun 2017, Adebayo, and Sun 2022). One effective strategy to mitigate this is the optimal placement of reactive power compensation devices, such as Static VAR Compensators (SVCs), to support system voltage levels during contingencies (Kheshti and Ding 2018, Salma, 2014). However, the effectiveness of such compensation largely depends on the accurate identification of weak or critical buses within the network that are most susceptible to voltage instability (Ahiakwo *et al.,* 2024, Alayande *et al.,* 2019). Thus, voltage stability assessment and bus sensitivity analysis are essential for reliable and resilient power system operation.

Several methods have been employed to identify these weak buses in power networks. These include modal analysis and load power margin techniques, continuation power flow (CPF) methods, and various stability indices such as the Voltage Stability Index (VSI), Fast Voltage Stability Index (FVSI), Line Stability Index (LSI), and Relative Electrical Distance (RED) (Chayapathy, 2016). For instance, Oluseyi *et al*. (2015) applied LSI and LSF to detect vulnerable lines in the Nigerian 31-bus system. Adedayo (2017) introduced a statistical approach based on the voltage standard deviation-to-mean ratio, while Idoniboyeobu *et al*. (2018) used a probabilistic index (Q-LVSI) for weak bus detection. Furthermore, techniques such as Adaptive Neuro-Fuzzy Inference Systems (ANFIS) and Coupling Strength Matrix (CSM) have also been applied for grid stability analysis and critical node identification. More recent works have introduced hybrid and intelligent models, such as the Quadratic Line Voltage Stability Index (Q-LVSI) and Adaptive Neuro-Fuzzy Inference Systems (ANFIS)-based approaches.

In addition, researchers have explored the use of FACTS devices to improve voltage stability. Ugwuanyi *et al*. (2024a) demonstrated the benefits of optimally placing FACTS devices to enhance renewable energy integration and grid stability. Similarly, Ozioko *et al*. (2019) employed STATCOM to improve transmission performance and dynamic voltage support. Another study by Ugwuanyi *et al*. (2024b) proposed a simplified method using STATCOM to simultaneously enhance both voltage and angle stability in Nigerian power networks. These studies underscore the growing interest in intelligent, device-based interventions, but they also highlight the ongoing need for improved methods of identifying critical buses under complex loading conditions.

Despite the effectiveness of these techniques, some suffer from limitations such as high computational complexity, slow convergence, and sensitivity to parameter changes, especially in large-scale networks. Others fail to adapt when voltage control devices or nonlinear constraints are involved. These drawbacks highlight the need for a more efficient and reliable index for identifying weak buses under different operating conditions.

To address these gaps, this paper proposes the development and application of a novel index termed the Reactive Power Loss Index (RPLI), formulated using the bus admittance matrix method. The RPLI quantifies the reactive power loss at load buses, allowing for the ranking of bus sensitivity to voltage instability. The method was tested on the Nigerian 52-bus 330 kV transmission system under contingency conditions, and its performance was benchmarked against RED and FVSI approaches..

METHODOLOGY

Research Approach

The goal of this study is to improve voltage stability in a power system during contingency by optimally identifying weak buses and determining suitable locations for reactive power compensation. This is formulated as a nonlinear constrained optimization problem, where the main goal is to determine the optimal location for reactive compensation devices and the cost in the power system. Hence, the objective function is to optimize the reactive power deficiency or surplus at the system load bus, given as Equations (1) to (4) (Marek, 2003, Ozioko *et al.,* 2019, Rosehart *et al*., 2002).

$$f = optimize \cdot \left(\left(\frac{(1-\sin\theta)V_{GS}^2}{2X\cos^2\theta} \right) - \left(\sqrt{\frac{V_{GS}^4}{4X} - Q_r \frac{V_{GS}^2}{X}} \right) \right) * CQ_{G,i} \tag{1}$$

$$S \frac{(1-\sin\theta)V_{GS}^2}{2X\cos^2\theta}_{max} \tag{2}$$

$$P\sqrt{\frac{V_{GS}^4}{4X} - Q_{Gr}\frac{V_{GS}^2}{X}}_{max} \quad (3)$$

$$CQ_{G,i} = Q_{G,i} \times C_G \quad (4)$$

where f is the objective function, $P_{max}$is the maximum active power, $S_{max}$is the maximum generator apparent power, $\theta$is the transmission line angle, $Q_{Gr}$is the reactive power. G is the generator bus Q_{Gi} is the reactive power contribution from the generator to the load bus i. C_G is the bid price of the generator in ($/MVAR), C is the economic metrics (transmission line charges). ($/Ω).

The optimization is subject to the following power system operational constraints given in Equations (5) to (9).

$$P_{Gi} - P_{Li} = |V_i| \sum_{j=1}^{N} |V_j| \left(G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij} \right) \quad (5)$$

$$Q_{Gi} - Q_{Li} = |V_i| \sum_{j=1}^{N} |V_j| \left(G_{ij} \sin\theta_{ij} + B_{ij} \cos\theta_{ij} \right) \quad (6)$$

$$Q_{Gi}^{min_{Gi} {}_{Gi}^{max}} \quad (7)$$

$$0.95p.u \le V_i \le 1.05p.u \quad (8)$$

$$|S_{ij}| \le 1.25 \cdot S_{ij}^{rated} \quad (9)$$

where; P_{Gi}, Q_{Gi} are active and reactive power generated at bus i, P_{Li}, Q_{Li} are active and reactive power load at bus i, V_i, V_j are voltage magnitudes at buses i and j, G_{ij}, B_{ij} are conductance and susceptance between buses i and j, θ_{ij} is the phase angle between buses i and j, S_{ij} is the apparent power flow through the transmission line connecting buses i and j, and the 125% limit reflects contingency tolerance.

Nigerian 52-bus 330 kV transmission system

The system under study is the Nigerian 52-bus, 330 kV transmission network shown in Figure 1. The network consists of seventeen (17) generation stations, thirty-five (35) load stations and fifty-three (53) transmission lines. All buses presented correspond to the 330 kV voltage level of the Nigerian transmission network. The system is modeled after the Nigerian national grid's 330 kV transmission backbone, which connects generation stations, major substations, and regional load centers across the country. National Control Centre (NCC) situated in Osogbo, Osun State, is saddled with the central responsibility of controlling the transmission system, with a backup NCC situated in Shiroro. The corresponding system load data shown in Table 1 required for this study was obtained from the National Control Centre (NCC) of the Transmission Company of Nigeria (TCN), Osogbo, Nigeria.

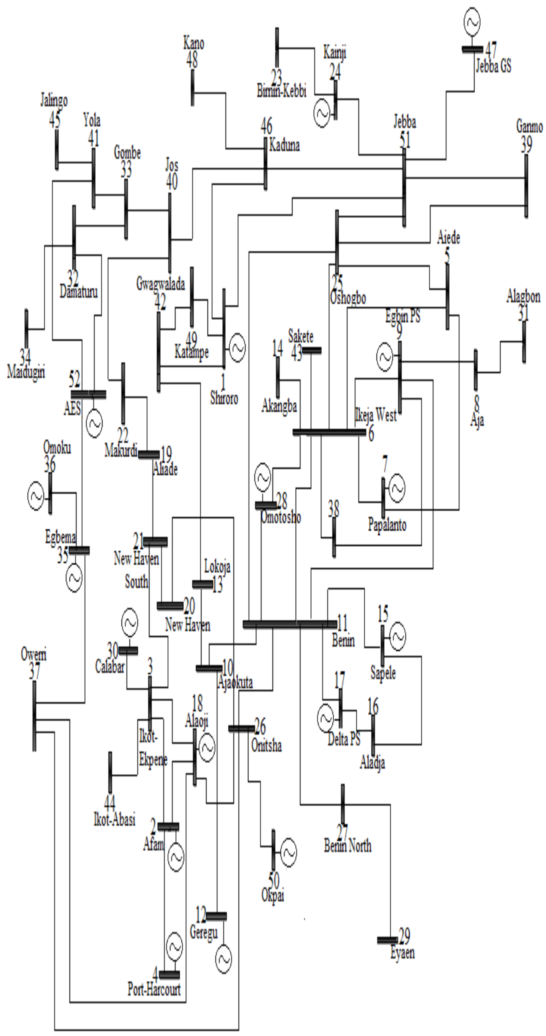

Figure.1:Nigerian 330 kV, 52-bus transmission network

Table 1: Load data of Nigerian 52-bus 330 kV transmission system

Bus No	Bus Type	Voltage Magnitude (p.u)	Phase Angle (Degree)	Load		Generator	
				P (MW)	Q (MVar)	P (MW)	Q (MVar)
1	Swing	1	0	0	0	480	240
2	PV	1	0	315	157.5	760	428
3	PV	1	0	321	160.5	578	207
4	PQ	1	0	316	158	0	0
5	PV	1	0	70.5	35.11	600	298
6	PQ	1	0	60.5	30.11	0	0
7	PQ	1	0	700	350	0	0
8	PQ	1	0	300	150	0	0
9	PQ	1	0	110	55	0	0
10	PV	1	0	230	115	414	207
11	PQ	1	0	360	80	0	0
12	PQ	1	0	75.1	37.5	0	0
13	PV	1	0	300	150	335	167.5
14	PQ	1	0	200	100	0	0
15	PQ	1	0	179	89.5	0	0
16	PV	1	0	315	157.5	1020	510
17	PQ	1	0	107.4	53.49	0	0
18	PV	1	0	65	33	882	441
19	PQ	1	0	136	84	0	0
20	PQ	1	0	72	45	0	0
21	PQ	1	0	39	27.8	0	0
22	PQ	1	0	84	50	0	0
23	PQ	1	0	146	84.5	0	0
24	PQ	1	0	32	17.8	0	0
25	PQ	1	0	110	80	0	0
26	PQ	1	0	100	58.4	0	0
29	PQ	1	0	440	220	0	0
30	PV	1	0	400	200	480	240
31	PQ	1	0	400	200	0	0
32	PQ	1	0	450	225	0	0
33	PQ	1	0	400	200	0	0
34	PV	1	0	440	220	931.6	465.8
35	PQ	1	0	400	200	0	0
36	PV	1	0	450	225	300	150
37	PQ	1	0	400	200	0	0
38	PQ	1	0	440	220	0	0
39	PV	1	0	400	200	500	250
40	PQ	1	0	450	225	0	0
41	PQ	1	0	440	220	0	0
42	PQ	1	0	400	200	0	0
43	PV	1	0	450	225	253	126.5
44	PQ	1	0	430	215	0	0

45	PQ	1	0	450	225	0	0
46	PQ	1	0	460	230	0	0
47	PV	1	0	450	225	600	298.8
48	PQ	1	0	460	230	0	0
49	PQ	1	0	480	240	0	0
50	PV	1	0	400	200	730	365
51	PQ	1	0	450	225	0	0
52	PV	1	0	440	220	500	250

Mathematical model and derivation of reactive power loss index

The Reactive Power Loss Index (RPLI) is developed as a voltage stability indicator by quantifying the share of reactive power loss attributed to each load bus, based on system operating conditions and network topology. A single line diagram of a power system is shown in Figure 2 using bus admittance matrix methods (Lavaei and Low, 2010).

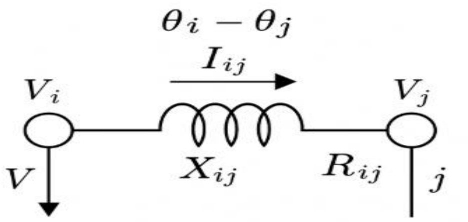

Figure 2: One-line diagram of transmission network

Its derivation starts from the bus admittance matrix formulation and applies power system fundamentals as follows. The network was modelled using the standard bus admittance matrix Y_{bus}, partitioned into generator (G) and load (L) buses as given in Equation (10).

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \cdot \begin{bmatrix} V_G \\ V_L \end{bmatrix} \quad (10)$$

where, I_G, I_L are the complex current injections at generator and load buses, V_G, V_L are the complex voltages at generator and load buses $Y_{GG}, Y_{GL}, Y_{LG}, Y_{LL}$ are the sub-matrices of the admittance matrix.

The bus admittance load and generator current injection vectors for the power system were calculated using Equations (11) and (12).

$$[I_L] = [Y_{LG}] \cdot [V_G] + [V_{LL}] \cdot [V_L] \quad (11)$$

$$[I_G] = [Y_{GG}] \cdot [V_G] + [V_{GL}] \cdot [V_L] \quad (12)$$

where: I_G, I_L are the complex bus current injection vectors.

The complex power at each load bus i is given by Equation (13).

$$S_{Gi} = V_{Gi} I_i^* = P_{Gi} + jQ_{Gi} \quad (13)$$

Equivalent shunt admittanceY_G of generator node was evaluated using Equation (14).

$$Y_{Gj} = \frac{1}{V_{Gj}} \left(\frac{-S_{Gj}}{V_{Gj}} \right)^* \quad (14)$$

The voltage contribution to generator bus was determined using Equation (15).

$$V_{Gj} = \sum_{i=1}^{NL} Y_{GL,ij}^{B} \times V_{Li} \quad (15)$$

Equations (14) and (15) enable the calculation of reactive power injection at each bus based on the corresponding bus voltage and network admittance parameters. The reactive power loss allocated to each bus are calculated using Equation (16).

$$Q_{Lossi} = \sum_{j=1}^{N} \left(Q_{Gij} - Q_{Li} \right) \quad (16)$$

where: $(*)$ is conjugate, $S_G$is the generator apparent power, $V_G$is the generator voltage, Y_{GL}^{B} is the generator load bus admittance voltage, Q_{Li} is the net reactive power load demand, $N$is number of buses, $Q_{Loss,i}$ is the normalised reactive power loss.

For the entire network, the total reactive power **loss** at each load bus was estimated by summing the line losses associated with that bus. To allow comparison and ranking of buses, the per-bus reactive power losses are normalized.

Then, the mathematical framework of Reactive Power Loss Index (RPLI) was formed from Equation (9) using weighted sum of normalized values of reactive power loss at each load buses as given in Equation (17).

$$RPLI = \frac{Q_{Loss,i}}{max(Q_{Loss,i})} + \sum_{j=1}^{NC} \left(\frac{Q_{Loss,ij}}{max(Q_{Loss,ij})} + NCSI_j \right) \quad (17)$$

where $max(Q_{Loss,i})$ is the maximum normalized reactive power loss, $Q_{Loss,ij}$ is the normalized reactive power loss, $max(Q_{Loss,ij})$ is the maximum normalized reactive power loss, $NCSI$ is normalised number of severe contingencies selected, j^{yh} is the most critical node of contingency.

Equation (10) was used as an indicator of the reactive power deficiency or surplus at the power system load bus. The bus with the highest RPLI was considered the weakest bus in the system and as the potential bus for the best location of the reactive compensation device for additional voltage support. In addition, the bus with the lowest RPLI was considered to be more stable and did not require any compensation device to support the voltage..

Power flow solution and voltage stability analysis

To analyze voltage stability under various operating conditions, power flow simulations were carried out using MATLAB R2021a, an open-source power system analysis toolbox. The Nigerian 52-bus transmission network was modelled using bus admittance matrix representation, and power flow computations were based on the Newton-Raphson (NR) method for improved convergence accuracy. The voltage stability of the transmission system under contingency was evaluated using RPLI formulated from bus admittance technique.

Power flow solution of Nigerian 52-bus system at steady state

The power flow of Nigerian 52-bus transmission system was performed to determine the voltage stability of the system under steady state conditions using bus admittance technique. The network consists of seventeen (17) generation stations, thirty-five (35) load stations and fifty-three (53) transmission lines (Oluseyi *et al.*, 2015). Each bus was modeled with standard parameters such as bus type (PQ or PV), base voltage, generation capacity limits, and load demand values. All transmission lines were modeled with series impedance (resistance and reactance) and shunt admittance. Figure 2 was conceptually based on this configuration, representing the single-line equivalent of a typical transmission corridor connecting a generator to a load bus.

The total connected system load used for the simulation at steady state was: active power (P): 4,361.8 MW and reactive power (Q): 3,575.6 MVar. These values represent the combined demand across all 35 load buses and were distributed proportionally based on realistic regional load profiles.

The system node current and voltage were calculated using Equations (18) and (19) (Adedayo, 2017, Danish, 2020).

$$I_{Bus} = [Y_{Bus}] \cdot V_{Bus} \quad (18)$$

$$V_{Bus} = [Y_{Bus}{}^{-1}] \cdot I_{Bus} = [Z_{Bus}] \cdot I_{Bus} \quad (19)$$

The voltage violations were monitored for any bus close to the voltage rating limit of ± 5% (0.95 to 1.0 p.u). Also the active and reactive power of the system was determined. The simulation was executed in MATLAB R2021a using a customized script that interfaces with MATPOWER functions. The system admittance matrix was calculated programmatically, and contingency cases were implemented as separate iterations within a loop structure, allowing automated injection of disturbances.

Voltage stability evaluation at contingency with RLPI

To test system robustness and voltage stability limits, a contingency scenario was introduced by artificially increasing the reactive load (Q) at each of the 35 load buses by 75% above their base case values. The rationale for this stress test is as follows:

a) It emulates worst-case reactive power demand, such as during peak demand or in scenarios involving high levels of motor starting, air-conditioning, or sudden renewable fluctuations.

b) A uniform increase across all load buses provides a controlled environment to identify systemic voltage weaknesses.

c) This contingency reveals voltage collapse-prone areas and allows for the evaluation of the RPLI under stressed conditions.

Mathematically, this is given as Equation (20).

$$Q_{Li}^{New} = Q_{LI}^{base} \times 1.75 \quad (20)$$

where Q_{Li}^{base} is the reactive power at the load bus at the base case (steady state).

The voltage stability of a given transmission system contingency condition was analysed using the RPLI formulated from the bus admittance matrix. The line MVA limits violation of the power system was checked. Then, the critical buses in the power system were identified and selected based on the value of the RPLI obtained.

Simulation was carried out based on the following step:

Step 1: Transmission power system data are inputted and the voltage magnitudes of the power system are initially set to zero p.u.

Step 2: The bus admittance matrices are formed and steady state operating conditions of the power system are determined using Equations (11) and (12).

Step 3: The reactive power demand of load buses from the base case is gradually increased by 75% for contingency analysis, and the Iteration count is set to 1.

Step 4: The equivalent shunt admittance of generator node are calculate using Equation (14), then, bus admittance matrix for contingency is formed.

Step 5: The voltage contribution to generator bus from each load bus voltage and the generator current at a contingency are determined using Equations (15) and (21).

$$I_G = \left[[Y_{GG}] - [Y_{GL}] \cdot [Y_{LL}]^{-1} \cdot [Y_{LG}]\right] \cdot [I_L] + [Y_{GL}] \cdot [V_L] \quad (21)$$

Step 6: The reactive load capacity (power contribution) are determined using Equation (22).

$$Q_{GLi} = \sum_{i=1}^{NL} Q_{Gij} \quad (22)$$

where; Q_{GLi} is the reactive power contribution, Q_{Gij} is the generator's reactive power

Step 7: The maximum load capacity of each bus is checked. If the values of the load capacity are more than the specified limit, the admittance matrix is modified, and step 3 is repeated. Else step 8 is executed.

Step 8: Reactive power loss allocated to each bus is calculated using Equation (10).

Step 9: The value of RPLI was evaluated, and the weakest buses were identified

Step 10: The process was stopped

Comparison and performance evaluation

In order to verify the complexity of the RPLI for identification of critical buses for the placement of a reactive power device on the power system during contingency, a Static Var Compensators (SVC) was installed on the selected critical load buses. Also, the performance of the RPLI on the Nigerian power system based on the voltage magnitude, system maximum loadability, reactive power loss and transmission charges was compared with the Relative Electrical Distance (RED) approach and Fast Voltage Stability Index (FVSI) used for the same purpose.

Implementation of RED

The RED approach generated from bus admittance matrix was used to identify the weak buses with a high impact of voltage instability in the Nigerian 52-bus transmission system based on the objective function in Equation (1). The RED is a voltage stability index used to evaluate the electrical proximity between load buses and generator buses in a power system. It quantifies how strongly a load bus is electrically connected to the power sources (generators) by analyzing the sensitivity of voltage changes in relation to reactive power support. A higher RED value implies a weaker connection to sources, indicating that the bus is more vulnerable to voltage instability during disturbances (Adebayo *et al.,* 2018, Chertkov *et al*., 2011).

This method is grounded in the principle that the effectiveness of voltage support at a load bus depends on its electrical distance from generator buses, which can be derived from the inverse of the Jacobian matrix or directly from the bus admittance matrix (Lavaei and Low 2010). The RED concept was used to compute the system reactive power from generator sources and switchable volt-amperes reactive (VAR) sources in the power system to meet the system load demands. Also, the transmission line charges' susceptances contribution to the system reactive flows are computed.

Given the bus admittance matrix, partitioned into generator and load buses, the current injections are expressed as Equation (10). The generator current and voltage at contingency are computed using Equations (23) and (24).

$$[I_G] = [Y_{GG}][V_G] + [Y_{GL}]\{[Y_{LL}]^{-1}[I_L] - [Y_{LL}]^{-1}[Y_{LG}][V_G]\} \qquad (23)$$

$$[V_L] = [Z_{LL}][I_L] + [F_{LG}][V_G] \qquad (24)$$

The desired generation proportions matrix are computed using Equation (25).

$$[D_{LG}] = abs\{[F_{LG}]\} \qquad (25)$$

The [RED] are compute from the $[D_{LG}]$matrix in Equation (25) as Equation (26)

$$[RED] = M - [D_{LG}] \qquad (26)$$

where$M$is the unity matrix of size$L \times G$, $G$is the number of generator buses and$L$ is the number of load buses, $[F_{LG}]$gives the relation between load bus and source bus voltages.

The significance of the approaches was compared with RPLI approach. Simulation of the approach was carried out in MATLAB R2021a.

Implementation of fast voltage stability index

The Fast Voltage Stability Index (FVSI) was computed from contingency calculation using bus admittance matrix. The FVSI was used to estimate the proximity of the transmission line to voltage collapse by evaluating the relationship between reactive power demand and the line's capacity to maintain stable voltage in the Nigerian 52-bus transmission system.

The FVSI between a sending bus i and a receiving bus j is given by Equation (27) (Adebayo and Sun

2017, Idoniboyeobu *et al*., 2018, Rosehart *et al.,* 2002).

$$FVSI_{ij} = \frac{4Q_jX_{ij}}{V_i^2\left(R_{ij}^2+X_{ij}^2\right)} < 1 \quad (27)$$

Where V_s is the voltage at sending end, Q_j reactive power load at the receiving bus j , X_{ij} is the series reactance of the transmission line between buses i and j, R_{ij} is the series resistance of the transmission line between buses i and j.

Simulation was carried out in MATLAB R2021a and a severity order of the load buses based on the values of FVSI was determined. The bus with stability index value closest to one (1) is the most critical bus and was considered as the weakest bus in the system and as the potential bus for best location of the SVC device reactive. The system maximum loadability and reactive power loss are calculated using Equations (3) and (14), respectively. In addition, the transmission line charges susceptances contribution to the system reactive flows was calculated.

Implementation of static var compensator

To assess the effectiveness of the proposed Reactive Power Loss Index (RPLI), Static VAR Compensators (SVCs) were integrated into the Nigerian transmission system at the identified critical buses by RPLI. The SVCs were modeled using a firing angle-based control approach, enabling the injection or absorption of reactive power depending on system conditions. These devices were implemented as variable reactance elements connected in shunt to the system and treated as reactive power support generators.

The SVC effective reactance X_{SVC} was determined using Equation (28) (Danish *et al*., 2020, Nor and Sulaiman, 2019).

$$X_{SVC} = \frac{X_CX_L}{\left[\frac{X_C}{\pi}\{2(\pi-\alpha)+Sin(2\alpha)\}-X_L\right]} \quad (28)$$

The minimum and maximum SVC reactive power is given as Equations (29) and (30) (Adebayo and Sun, 2021, Ozioko *et al*., 2019).

$$Q_{SVC}^{max}\frac{-V_j^2}{X_CX_L}\left\{X_L-\frac{X_C}{\pi}[2(\pi-\alpha)+Sin(2\alpha)]\right\} \quad (29)$$

$$Q_{SVC}^{min}\frac{-V_j^2}{X_C}\left\{-\frac{X_C}{\pi}[2(\pi-\alpha)+Sin(2\alpha)]\right\} \quad (30)$$

At $Q_{SVC} < 0$, SVC injects reactive power into the network while

At $Q_{SVC} > 0$, SVC absorb reactive power from the network.

At the end of the iteration, the variable firing angle α and generated voltage are updated using Equations (31) and (32) (Ugwuanyi *et al.,* 2024a, Ugwuanyi *et al*., 2024b)

$$\alpha_{SVC}^{(i)} = \alpha_{SVC}^{(i-1)} + \Delta\alpha_{SVC}^{(i)} \quad (31)$$

$$V_{ij} = V_{ref} + X_{SVC}I_{SVC} \quad (32)$$

Where B_{SVC} is the susceptance of SVC , V_{ij} is the generator voltage, V_j is the system voltage at bus j, P_{Gj} is the generator active power of SVC at bus j, Q_{Gj} is the generator reactive power of SVC at bus j., θ_j is the angle at bus at bus j, i and (i-1) denote previous and next iteration, respectively. I_{SVC} is the control bus SVC current, Q_{SVC} is the reactive power drawn by SVC, α is the firing angle.

The changes in variable firing angle value of the SVC are used to maintain the nodal voltage magnitude at the selected buses.

RESULTS AND DISCUSSION

Figure 3 presents the voltage variation of the power system at steady state. It was observed that the Nigerian 52-bus system at steady state analysis shows a stable system, as all the bus voltages of the power system met the acceptable voltage range of 0.95 to 1.05 p.u. This justified that the power system is one of the improved Nigerian transmission systems (Oluseyi *et al.,* 2015).

Table 2 presents the selected buses of the power system at contingency (75% increase in reactive base load). It could be observed that the load bus voltages of the power system start to decrease due to insufficient power generation and these resulting in an unstable system condition which can cause voltage collapse in the power system. Buses 4, 6, 7, 8, 11, 12, 17, 19, 20, 21, 22, 23, 27, 29, 32, 33, 35, 38, 44, 45, 46 and 51 with voltage magnitude of 0.9384, 0.9250, 0.9449, 0.9210, 0.9350, 0.9490, 0.8085, 0.8010, 0.9272, 0.9242, 0.9415, 0.9404, 0.9436, 0.9285, 0.8701, 0.8686, 0.8086, 0.9245, 0.9275, 0.9330, 0.8045 and 0.9420 p.u, respectively, are buses whose voltage falls short of the voltage working range of □5 and the line MVA limits of 125%.

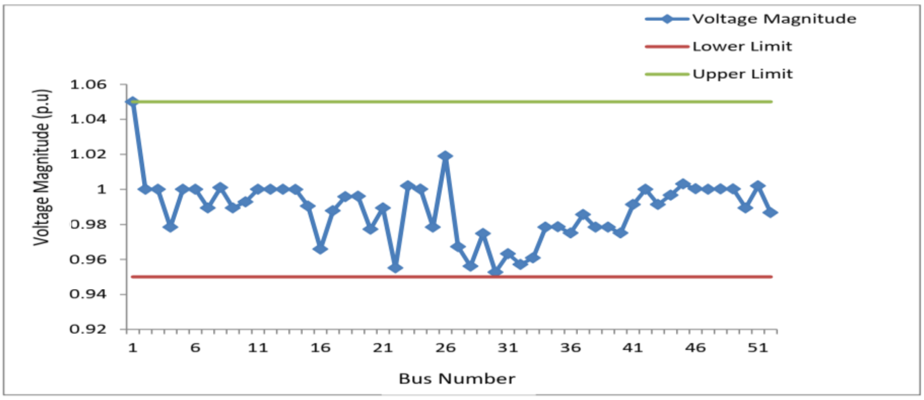

Figure 3: Voltage variation for the Nigerian 52-bus system at steady state

Table 2: Selected bus for the Nigerian 52-bus system at contingency

Buses		Voltage Magnitude	Maximum Loading	Power Loss
No	Type	(p.u)	P(MW)	Q(MVar)
4	PQ	0.9384	338.6	4.81
6	PQ	0.9250	75.3	81.45
7	PQ	0.9449	704	24.08
8	PQ	0.9210	398.1	16.24
11	PQ	0.9350	432	33.55
12	PQ	0.9490	101.2	7.5
17	PQ	0.8085	125.3	21.37
19	PQ	0.8010	164.8	7.26
20	PQ	0.9272	86.9	11.03
21	PQ	0.9242	52.3	1.38
22	PQ	0.9415	98,5	22.44
23	PQ	0.9404	185.4	3.73
27	PQ	0.9436	97.4	1.49
29	PQ	0.9285	613	2.92
32	PQ	0.8701	632	21.56
33	PQ	0.8686	532	0.25
35	PQ	0.8086	545	8.39
38	PQ	0.9245	620	9.12

44	PQ	0.9275	620	14.58
45	PQ	0.9330	620	9.51
46	PQ	0.8045	545	33.22
51	PQ	0.9420	545	9.76

From Table 2, the selected buses have a maximum loadability of 338.6 MW; 75.3MW; 704 MW; 398.1MW; 432MW; 101.2MW; 125.3 MW; 164.8 MW; 86.9 MW; 52.3MW; 98,5MW; 185.4 MW; 97.4MW; 613MW; 632MW; 532MW; 545MW; 620MW; 620MW; 620MW; 545MW; 545MW, respectively.

Table 3 presents the results of selected load buses at contingency based on the application of RPLI. The buses with the highest value of the RPLI were considered the weakest buses in the system for additional voltage support and were ranked accordingly. It could be observed that buses 17, 19, 32, 33, 35 and 46 were buses that have the highest value of RPLI in the power system. These buses have reactive power loss values of 21.37, 42.49, 22,56, 3.25, 8.39, 32.62 MVar and were ranked 4, 1, 3, 6, 5 and 2, respectively. Also, the transmission line charges of these buses based on the line resistance are 13.61, 116.93, 82.30, 3.85, 11.56 and 99.04 $/Ω, respectively. The results indicate that the bus transmission charges increase as the RPLI value increases.

Table 3: Selected bus for the Nigerian 52-bus power with RPLI

	Voltage	**Maximum Loading**	**RPLI**		**Transmission line Charges**
Bus No	**Magnitude (p.u)**	**P(MW)**	**Q(MVar)**	**Ranking**	**($/Ω)**
17	0.7054	120.3	21.37	4	13.61
19	0.7000	140.8	42.49	1	116.93
32	0.7200	630	22.56	3	82.30
33	0.7660	530	3.25	6	3.85
35	0.7061	540	8.39	5	11.56
46	0.7051	540	32.62	2	99.04

In addition, Figure 4 illustrates a comparison of the total active and reactive power losses under three scenarios: steady-state, contingency, and contingency with the application of the RPLI approach. At steady state, the total active and reactive power losses were 113.87 MW and 134.57 MVar, respectively. Under contingency conditions, these losses increased significantly to 141.93 MW (a 24.6% rise) and 177.03 MVar (a 31.6% rise). However, with the application of RPLI during contingency, the total losses were reduced to 140.85 MW and 165.03 MVar—representing reductions of 0.76% and 6.78% in active and reactive power losses, respectively, when compared to the contingency-only scenario. These results demonstrate the effectiveness of the RPLI approach in reducing power losses under stressed operating conditions.

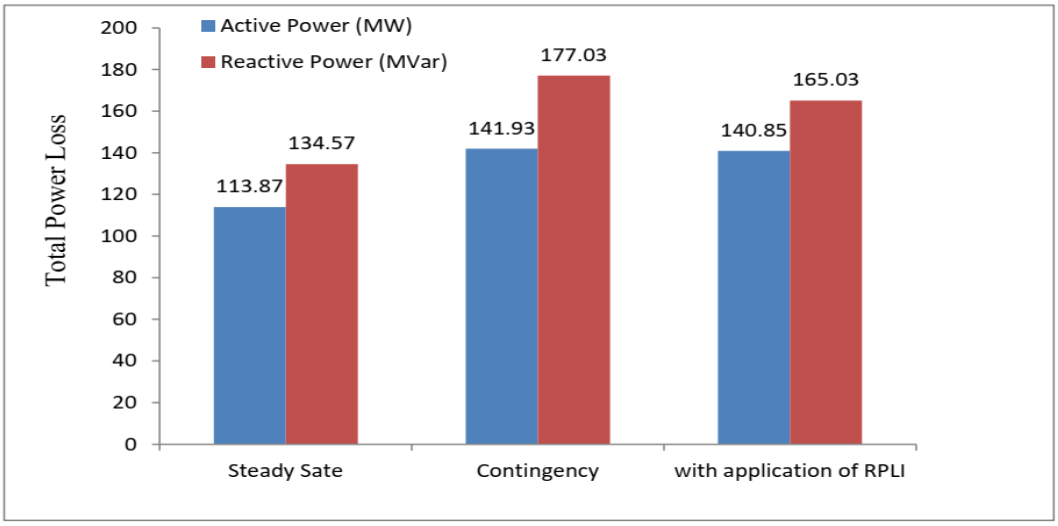

Figure 4: Comparison of Total power loss of Nigerian 52-bus with RPLI modern power systems

Figure 5 shows the comparison of voltage magnitude of the selected buses 17, 19, 32, 33, 35 and 46 in the power system with the application of RPLI, RED and FVSI at contingency. The buses are potential buses for optimal placements for compensation devices in the power system. It can be observed that the voltage magnitudes of these buses with application of RPLI are 0.7054, 0.7000, 0.7200, 0.7660, 0.7061 and 0.7051 p.u., respectively. With applications of RED and FVSI approaches, the voltage magnitudes of these buses are 0.9554, 0,9500, 0.9200, 0.9660, 0.9561 and 0.9551 p.u., respectively; 0.7050, 0.7020, 0.7202, 0.7060, 0.7014 and 0.7250 p.u., respectively. It was observed that the application of RED provides a more stable voltage magnitude than RPLI and FVSI.

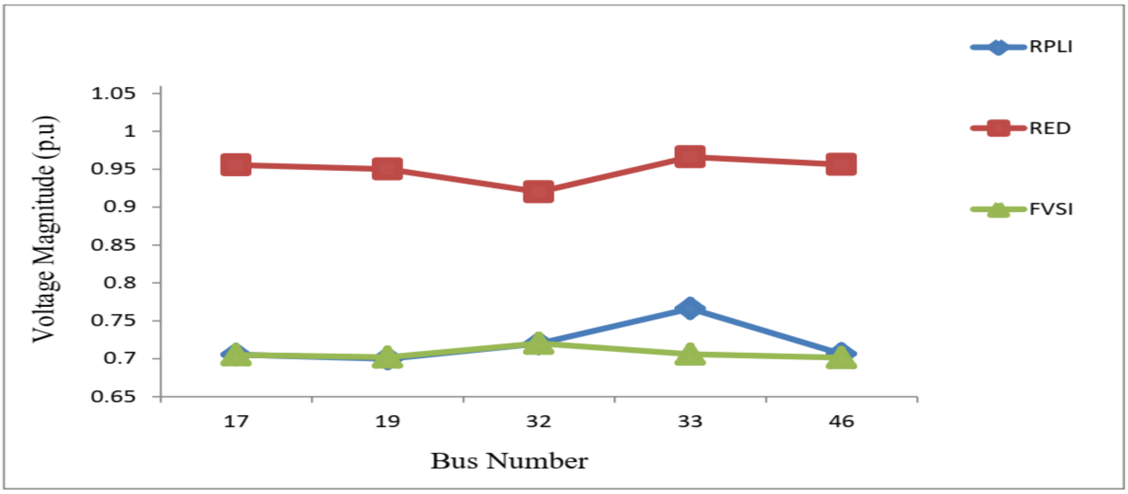

Figure 5: Comparison of Voltage Magnitude of Nigerian 52-bus at contingency

Figure 6 presents the comparison of the maximum loadability of the selected buses with the application of RPLI, RED and FVSI at contingency. With application of RPLI, the selected load buses have maximum loadability value of 120.3, 140.8, 630, 530, 540 and 540 MW, respectively. With application of RED and FVSI, the maximum loadability of the selected buses is 126, 142, 636, 538, 545 and 545 MW; 126, 140, 630, 530, 540 and 540 MW, respectively. It could be observed that the results of the approaches are varying with each other.

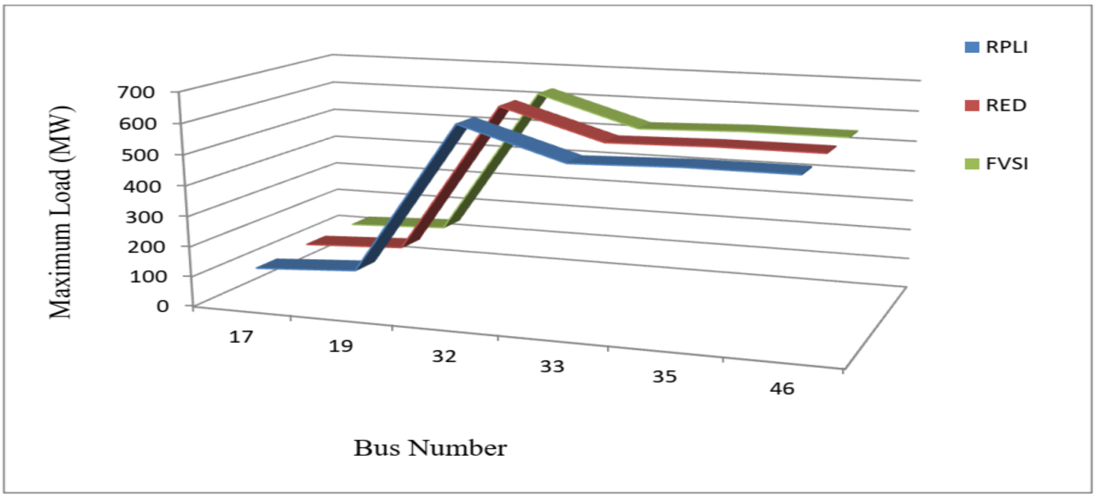

Figure 6: Comparison of maximum load of Nigerian 52-bus system

Also, Figure 7 illustrates the comparison of reactive power loss of the selected load buses with the application of RPLI, RED and FVSI at contingency. The reactive power loss of the selected load buses with application of RPLI is 21.37, 42.49, 22,56, 3.25, 8.39 and 32.62 MVar compared with the value obtained with application of RED and FVSI of 21.37, 42.49, 22.56, 0.25, 8.39 and 32.62 MVar, each respectively.. It could be observed that the reactive power loss with applications of RPLI and FVSI gave better results when compared with the application of RED.

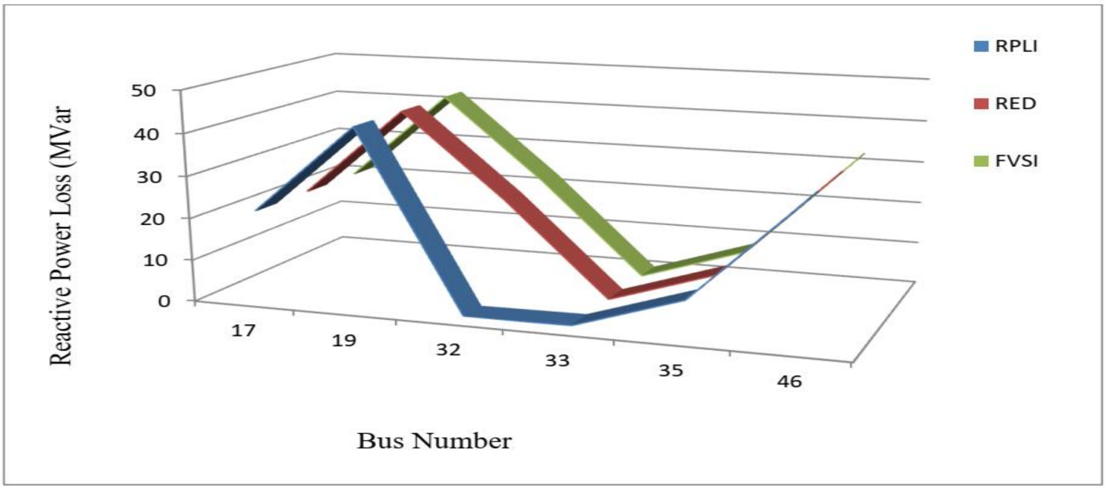

Figure 7: Comparison of reactive power loss of Nigerian 52-bus system

Table 4 provides a comparative assessment of the transmission line charges (in $/Ω) for six critical buses 17, 19, 32, 33, 35, and 46 under contingency conditions using three different methods: RPLI, RED, and FVSI. These charges represent the economic impact associated with reactive power flow and losses on the transmission lines linked to the selected buses. The purpose of this comparison is to evaluate whether the buses identified for compensation not only ensure technical stability but also minimize associated operational costs. The results show that RPLI produces transmission charges that are relatively moderate and consistent across all selected buses. For instance, at Bus 33, RPLI yields a significantly lower cost (3.85 \$/Ω) compared to RED (118.47 \$/Ω) and FVSI (116.71 \$/Ω), suggesting that RPLI is more effective in identifying compensation points with lower economic burden.

Table 4: Comparison of transmission charges for the 52-bus power

Buses		RPLI Transmission Charge (\$/Ω)	RED Transmission Charge (\$/Ω)	FVSI Transmission Charge (\$Ω)
No	**(\$/Ω)**			
17	PQ	3.85	4.03	4.03
19	PQ	11.56	11.66	11.58
32	PQ	99.04	99.98	99.04
33	PQ	116.9	118.47	116.71
35	PQ	13.61	13.73	13.61
46	PQ	82.30	83.06	82.30

The data in Table 4 highlight the advantage of using RPLI for selecting optimal compensation locations by balancing voltage stability needs with economic considerations. While RED tends to identify buses with higher voltage magnitudes, it may overlook the associated cost implications, leading to higher line charges. FVSI shows results closer to RPLI but with more variability across buses. In contrast, RPLI consistently selects buses that not only require voltage support but also offer cost-effective compensation opportunities. This reinforces the practical applicability of RPLI as a robust and economically efficient tool for voltage stability assessment and planning in real-world power systems.

Thus, it can be validated that the application of RPLI is suitable for selecting the best placement for SVC compensation devices in an electrical power system during contingency, as shown in Table 5.

Table 5: Comparison of power system performance with RPLI and incorporation of SVC at selected critical buses

Bus No.	Voltage Magnitude (p.u)		Max Loadability (MW)		Transmission Charges (\$/Ω)	
	RPLI	With SVC	RPLI	With SVC	RPLI	With SVC
17	0.7054	1.0000	120.3	108	13.61	11.01
19	0.7000	1.0000	140.8	1.34	116.93	90.28
32	0.7200	1.0000	630	450	82.30	67.06
33	0.7660	1.0000	530	410	3.85	3.86
35	0.7061	1.0000	540	400	11.56	10.08
46	0.7051	1.0000	540	440	99.04	70.74

From the above results, RPLI offers improved robustness and practical sensitivity to reactive losses, making it well-suited for identifying optimal SVC compensation points in real-time or planning contexts. While RED may present optimistic voltage levels and FVSI captures dynamic line behaviour, RPLI delivers a balanced and loss-focused perspective, ensuring technical and economic efficiency during system stress. These advantages confirm that RPLI is not only a viable alternative to existing indices but a potentially superior choice for voltage stability assessment and reactive power planning in modern power systems.

An appropriate size of 8 kVar SVC device was incorporated on the selected critical buses 17, 19, 32, 33, 35 and 46 with RPLI to improve the stability of the power system. Table 5 illustrates the comparison of voltage magnitude of the power system with RPLI and incorporation of SVC at contingency. It can be observed that the voltage magnitude of these buses was improved to 1.0000 p.u. compared with the value of RPLI values of 0.7054, 0.7000, 0.7200, 0.7660, 0.7061 and 0.7051 p,u, respectively. Also, it could be observed that the selected buses has maximum loadability value of 108, 134, 450, 410, 400 and 440 MW with transmission charges value of 11.01, 90.28, 67.06, 3.46, 10.08 and 70.74 \$/Ω, respectively compared with RPLI value of 120.3, 140.8, 630, 530, 540 and 540 MW with 13.61, 116.93, 82.30, 3.85, 11.56 and 99.04 \$/Ω; respectively. In addition, the total active and reactive power loss in the power system are reduced to 107.98 MW (23.9%) and 124 MVar (29.9%) compared with the contingency value of 141.93 MW and 177.03 MVar, respectively. Also, when compared the total active and reactive power loss with application of RPLI, the total active and reactive power loss reduced by 32.87 MW (23.3%) and 41.03 MVar (24.9%), respectively

The results clearly illustrate how the incorporation of appropriately sized 8 kVar SVC devices on RPLI-selected critical buses significantly improves voltage stability, reduces power losses, and lowers transmission costs, confirming the effectiveness of the proposed approach.

The key contributions of this study center around the development and application of a novel voltage stability assessment tool known as the Reactive Power Loss Index (RPLI). Unlike conventional indices, RPLI is derived from the bus admittance matrix and offers a normalized quantification of reactive power losses across load buses, providing a direct and comprehensive view of system vulnerability. This approach enables fast, accurate, and computationally efficient identification of weak buses, which is crucial for the optimal placement of reactive power compensation devices during contingency conditions. Simulation results on the Nigerian 52-bus 330 kV transmission system confirm that RPLI either matches or outperforms existing methods such as the Reactive Energy Distance (RED) and Fast Voltage Stability Index (FVSI), particularly by reducing total reactive power losses by 6.78% and active power losses by 0.76% under stressed operating conditions. The study thus delivers a context-specific solution tailored to the Nigerian grid, addressing its unique stability challenges while offering valuable insights for improving overall system reliability. Furthermore, a thorough benchmarking analysis highlights RPLI's superior performance in terms of voltage magnitude stability, system loadability, reactive losses, and economic factors such as transmission line charges. Collectively, these contributions establish RPLI as a novel, reliable, and practical voltage stability index, well-suited for real-world applications in complex and under-resourced power systems like Nigeria.

CONCLUSION

This study presented the development and application of a novel voltage stability assessment technique, the Reactive Power Loss Index (RPLI), aimed at identifying critical buses for optimal placement of reactive power compensation devices in the Nigerian 52-bus 330 kV transmission system. The RPLI was formulated using the bus admittance matrix to quantify reactive power losses at load buses under steady-state and contingency conditions. Simulation results showed that RPLI effectively identified weak buses with high vulnerability to voltage instability, outperforming or matching existing methods such as Relative Electrical Distance (RED) and Fast Voltage Stability Index (FVSI) in terms of accuracy, loss sensitivity, and cost-effectiveness.

Compared to RED and FVSI, RPLI provided more consistent and technically insightful results by directly associating reactive power loss with bus criticality, rather than relying solely on voltage magnitude or line-based parameters. Although RED yielded higher voltage magnitudes, it was less effective in minimizing reactive losses and often identified buses with higher transmission costs. RPLI demonstrated superior performance in maintaining system stability under contingency, with reduced active and reactive power losses and more economical transmission line charges.

The integration of 8 kVar SVCs at RPLI-identified buses further improved system performance. Voltage magnitudes at all selected buses were restored to 1.0000 p.u., and overall system losses were significantly reduced—active power loss decreased by 23.9% and reactive power loss by 29.9% compared to the contingency-only scenario. These improvements validate the practicality and robustness of the RPLI approach for both technical and economic optimization of reactive power support in real-world grid operations.

Thus, RPLI offers a reliable, computationally efficient, and cost-aware method for reactive power planning and voltage stability enhancement. Its integration with SVC placement strategies makes it a valuable tool for modern transmission system operators, particularly in developing power systems like Nigeria's. Future work will explore the application of RPLI in dynamic stability scenarios and its extension using intelligent optimization algorithms for automated compensation planning.

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