ANALYTICAL DESIGN OF CGPC-BASED PID CONTROLLERS FOR COMMONLY ENCOUNTERED ENGINEERING SYSTEMS

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ABSTRACT
The traditional PID controllers have continued to be the most widely implemented control technique in the industries for many years because of its structural simplicity and its transparent tuning rules. In this work, an analytical design method for PID controllers based on continuous generalized predictive control (CGPC) law is proposed. The design method consists of two steps. The first step entails tuning a CGPC system to obtain a satisfactory closed-loop response. Thereafter a truncated Maclaurin series is employed to approximate the designed CGPC law. Four simulation examples are used to demonstrate the effectiveness of the proposed method. The four examples used are the commonly encountered engineering systems which range from a first order plus time delay system, a second order plus time delay system, a third order system and a non-minimum phase system which has been known to be very problematic to control. The simulation results obtained showed that the proposed CGPC-based PID controllers provided good set-point tracking and disturbance rejection and compared favourably well with PID controllers designed by Ziegler-Nichols and Direct-Synthesis methods. The controllers are also found to be robust as indicated by the small values of computed sensitivity peak.

Keywords: CGPC, PID controller, Direct Synthesis (DS), Ziegler-Nichols (ZN), Tyreus-Luyben (TL)

1. INTRODUCTION

Despite the wide development of advanced control methods, the PID controllers are still commonly used in industry for its structural simplicity and design rules of thumb. The control design problem of this research work involves the approximation of the continuous time version of the generalised predictive control (GPC) algorithm - Continuous time generalised predictive control (CGPC) by a proportional-integral-derivative (PID) controller for the control of commonly encountered engineering systems. Since Clarke *et al.* (1987)derived the formula for the GPC controller in discrete form and Demircioglu and Gawthrop (1988) derived its continuous time version, many new predictive control methods based on the GPC approach have been presented (Clarke and Scattolini, 1991; Demircioglu and Gawthrop, 1991; Demirciouglu, 1991) and certain aspects of theoretical analyses of the stability and robustness of the GPC can be found (Wellsted and Zarrop, 1991; Camacho and Bordons, 1991; Deng *et al*., 2003; Osunleke, 2010). Several successful applications in the chemical industry processes have been reported, which have clearly highlighted the merits of the method. Very recently, Osunleke (2010) incorporated an anti-windup and disturbance rejection capability scheme known as robust anti-windup generalised predictive control into the Demircioglu and Gawthrop algorithm.

Existing works have shown that a discrete version of a generalised predictive control (GPC) based PID has been derived. The different authors have used different methods in achieving these objectives. Cheng *et al.* (2003) presented a GPC-based PID with a cost function different from the known GPC cost function to include the proportional, integral, and derivative constants for the PID tuning. Miklovicova and Mrosko (2003) presents a GPC-based PID controllers design by comparing the GPC closed loop with the PID closed loop, deriving the conditions for equivalence to obtain the tuning constants. All these GPC-PID designs have only been presented in the discrete time domain.

This work therefore proposes the parameterization of PID controllers from CGPC control law using truncated Maclaurin series, an approach that will henceforth be called CGPC-based PID controllers. Four simulation examples will be used to illustrate how this method can be used to tune PID controllers. It will then be compared with other popular classical PID controller tuning rules such as Ziegler and Nichols (1964) and Direct synthesis methods to verify the effectiveness of the proposed system.

2. THE CGPC DESIGN ALGORITHM

Various predictive control algorithms differ in the model used for prediction. The CGPC algorithm uses transfer function model in the form expressed in equation (1) for prediction purposes;

$$Y(s) = \frac{B(s)}{A(s)}U(s) + \frac{C(s)}{A(s)}V(s) \qquad (1)$$

$A(s)$, $B(s)$, $C(s)$ are polynomials in Laplace operator, s, with degree n, m, n-1, respectively. Y, U, and V are the system's output, input, and disturbance input, respectively. C is usually chosen as a design polynomial having all roots in the left half plane.

The detail of CGPC algorithm will not be derived here as that is available in Demircioglu and Gawthrop(1991)or in its anti-wind up scheme in Osunleke (2010).

The CGPC control law can be obtained by minimizing the cost function;

$$J = \int_{T_1}^{T_2} [y_r^*(t,T) - w*(t,T)]^2 dT + \lambda \int_0^{T_2-T_1} [u_r^*(t,T)]^2 dT \qquad (2)$$

where

$$y_r^*(t,T) = y^*(t+T) - y(t) \qquad (3)$$

$$u_r^*(t,T) = \sum_{k=0}^{N_u} u_k(t)\frac{T^k}{k!} \qquad (4)$$

T_1= minimum prediction horizon

T_2= maximum prediction horizon

λ= control weighting

N_u =control order

Note thatT_1, T_2, λ and N_u are all tuning parameters; they can be used to achieve the desired closed-loop response.

The minimization of J in equation (2) results in the control law:

$$U(s) = g[W(s) - Y(s)] - \frac{G_c(s)}{C(s)}U(s) - \frac{F_c(s)}{C(s)}Y(s) \qquad (5)$$

Where g is a scalar gain, F_c and G_c are polynomials.

The feedback structure of this CGPC control law as given by equation (5) is illustrated in Figure 1

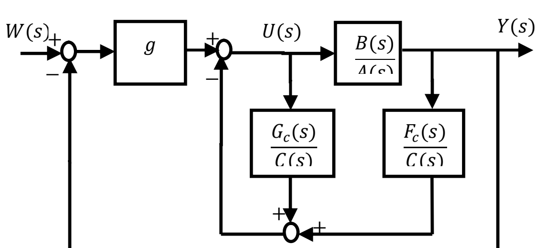

Fig. 1: The feedback system of CGPC

3. DERIVATION OF PI/PID CONTROLLER FROM CGPC CONTROL LAW

The block diagram in Fig. 1 for the implementation of CGPC control law in a closed loop system can easily be converted into a conventional feedback system shown in Figure2 through block diagram algebra and transformation, thus giving:

$$g_{CGPC}(s) = \frac{gA(s)C(s)}{A(s)C(s) + G_c(s)A(s) + F_c(s)B(s)} \qquad (6)$$

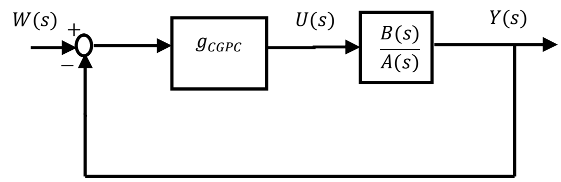

Fig 2: The conventional feedback control system

If the derived CGPC control law above is approximated by PID controller, we have

$$g_{CGPC}(s) \cong G_{PID}(s) = \left(\frac{k_D s^2 + k_C s + k_I}{s}\right) \qquad (7)$$

$$s \times g_{CGPC}(s) \cong k_D s^2 + k_C s + k_I$$

Let $f(s) = s \times g_{CGPC}(s)$

Thus

$$f(s) = k_D s^2 + k_C s + k_I \qquad (8)$$

By expanding f(s) into a Maclaurin series up to the second term, we have

$$f(0) + f'(0)s + 0.5f''(0)s^2 = k_I + k_C s + k_D s^2 \qquad (9)$$

By equating coefficients, we have

$$k_I = f(0) \qquad (10)$$

$$k_C = f'(0) \qquad (11)$$

$$k_D = 0.5f''(0) \qquad (12)$$

4. SIMULATION RESULTS

Four simulation examples are used to demonstrate the effectiveness of the proposed CGPD-based PID controller. In addition to designing PID controllers based on CGPC control law for each of the examples considered, the controllers were also designed using Direct Synthesis (DS), Internal Model Control (IMC), Ziegler-Nichols (ZN) and Tyreus-Luyben tuning rules.

For comparison purposes, the following robustness and performance metrics were used for assessment.

Robustness Metric: The peak value of the sensitivity function, Ms, is chosen as a measure of system robustness. This has been used widely by various researchers (Chen and Seborg, 2002). Recommended values of Ms are typically in the range of 1.2 – 2.0.

Performance Metrics: Two indices were used to evaluate controller performance. The integrated absolute error (IAE) is defined as:

$$IAE = \int_0^{\infty} |r(t) - y(t)|\, dt \qquad (13)$$

The total variation of the manipulated input u is used to evaluate the required control effort. The total variation has been widely acclaimed as a good measure of the "smoothness" of a control signal, and it should be as small as possible. It is given by:

$$TV = \sum_{k=1}^{\infty} |u(k+1) - u(k)| \qquad (14)$$

4.1 Example 1: First Order Plus Time Delay System

Consider the processdescribed by the transfer function

$$G_p(s) = \frac{e^{-s}}{s+1} \qquad (15)$$

Using first order pade approximant, we have

$$G_p(s) = \frac{-s+8}{s^2+9s+8} \qquad (16)$$

The disturbance model for CGPC algorithm is chosen as$C(s) = s + 1$, and the tuning parameters are selected as

T_1=0,T_2=1,P=1,N_y=3,N_u=1.5. The CGPC control law is obtained as:

$$G_{CGPC}(s) = \frac{5s^3 + 50s^2 + 85s + 40}{s^3 + 19s^2 + 18s} \qquad (17)$$

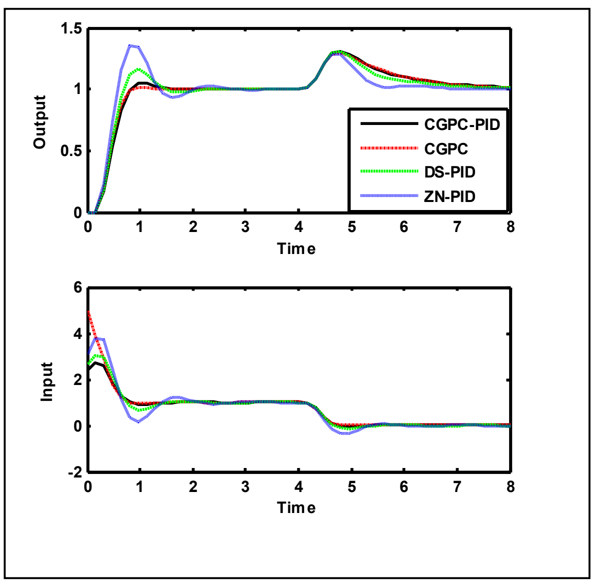

Fig. 3: Simulation results for Example 1

CGPC-PID controller was then obtained from (14) using equations (9) – (12).

The controller parameters and the performance metrics computed for CGPC law, CGPD-PID, DS-PID and ZN-PID are summarised in Table 1. All the controllers were found to be robust as measured by Ms which is less than 2 for all of them.

Figure 3 shows the simulation results obtained as a result of introducing unit step changes in the set point (as t=0) and in the disturbance (at t=4 sec). CGPC-PID approximated well the CGPC control law and provides a faster response than DS-PID and ZN-PID, with smoother control signal. The DS-PID and ZN-PID responses are a little bit oscillatory. They however provide faster responses to disturbance than CGPC-PID. The computed IAE and TV values corroborate all these claims.

4.2 Example 2: Second Order Plus Time Delay System

Consider a second order plus time delay system described in Seborg*et al.*(2004).

$$G_p(s) = \frac{2e^{-s}}{(10s+1)(5s+1)} \qquad (18)$$

Using optimal model reduction (Taiwo, 1999; Osunleke *et al.*, 2007), equation (18) is approximated to

$$G_p(s) = \frac{2}{57s^3 + 65s^2 + 16s + 1} \qquad (19)$$

The disturbance model is chosen as $C(s) = 0.5s^4 + s^3 + s^2 + s + 1$, and the tuning parameters selected as

T_1=0, T_2=1,P=1,N_y=3,N_u=1.5.

The CGPC control law is obtained as

$$C_{CGPC}(s) = \frac{33.7s^6 + 145s^5 + 243.5s^4 + 265.8s^3 + 199.7s^2 + 44.5s + 2.7}{0.5s^6 + 3.4s^5 + 11.1s^4 + 16.5s^3 + 15.5s^2 + 10.8s} \qquad (20)$$

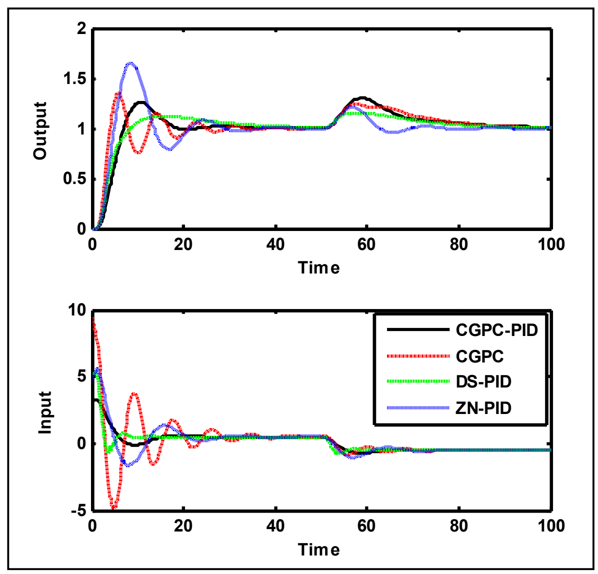

Fig. 4: Simulation results for Example 2

CGPC-PID controller was then obtained from (17) using equations (9) – (12).

The controller parameters and the performance metrics computed for CGPC law, CGPC-PID, DS-PID and ZN-PID are summarised in Table 2. CGPC law, DS-PID and CGPC-PID were found to be robust while ZN-PID was clearly not.

Figure 4 shows the simulation results obtained as a result of introducing unit step changes in the set point (as t=0) and in the disturbance (at t=50sec). CGPC-PID approximated well the CGPC control law and provide a faster response than DS-PID and ZN-PID, with smoother control signal though with a little bit of overshoot. The ZN-PID response is quite oscillatory. CGPC-PID however degraded a little bit in terms of disturbance rejection. The computed IAE and TV values corroborate all these claims.

4.3 Example 3: Non-Minimum Phase System

Consider the non-minimum phase system taken from Bequette

$$G_p(s) = G_d(s) = \frac{-9s + 1}{(15s + 1)(3s + 1)} \qquad (21)$$

The disturbance model for the CGPC algorithm is chosenas$C = s + 1$, while the tuning parametersare chosen as

T_1=8,T_2=25,P=1,N_y=20,N_u=0.5

The CGPC control law is obtained as

$$G_{CGPC}(s) = \frac{89.98s^3 + 123.17s^2 + 37.15s + 1.96}{46.71s^3 + 83.26s^2 + 37.31s} \qquad (22)$$

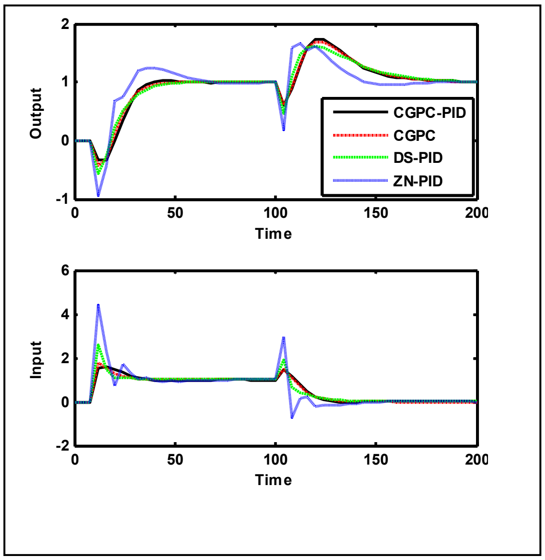

Fig. 5: Simulation Result for Example 3

CGPC-PID controller was then obtained from (19) using equations (9) – (12).

The controller parameters and the performance metrics computed for CGPC law, CGPD-PID, DS-PID and ZN-PID are summarised in Table 3. All the controllers were found to be robust as measured by Ms which is less than 2 for all of them.

Figure 5 shows the simulation results obtained as a result of introducing unit step changes in the set point (as t=0) and in the disturbance (at t=100 sec). CGPC-PID approximated well the CGPC control law and provide a faster response than DS-PID and ZN-PID, with smoother control signal. ZN-PID however performed better than all of them in terms of disturbance rejection. The computed IAE and TV values support all these claims.

4.4 Example 4: Laboratory-Scale Experimental Three Tank System

Consider a laboratory scale experimental three tank system housed in the process systems engineering laboratory.

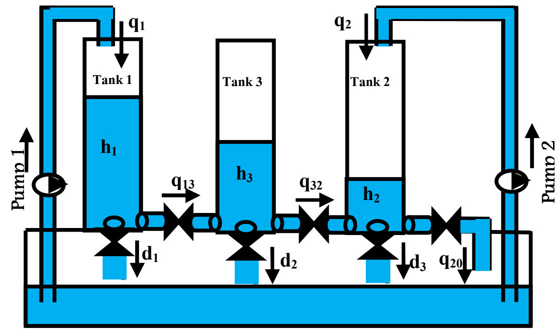

Fig. 6: Laboratory-Scale Experimental Three-Tank-System

The Three tank system is originally designed as a two-inputs two outputs system. With pump 2 turned off and level 2 considered as the only controlled variable, the system can be considered as a Single input single output system. The process model is thus obtained as:

$$G_p(s) = \frac{\delta h_2}{\delta q_1} = \frac{0.216}{5.55 \times 10^5 s^3 + 3.42 \times 10^4 s^2 + 497.39s + 1} \qquad (23)$$

The disturbance model for the CGPC algorithm is chosen as$C = s^4 + s^3 + s^2 + s + 1$, and the tuning parameters are T_1=0, T_2=500,P=1,N_y=12,N_u=6.

The CGPC control law is obtained as

$$G_{CGPC}(s) = \frac{88.4s^6 + 93.9s^5 + 93.9s^4 + 93.9s^3 + 5.5s^2 + 0.08s + 0.00016}{s^6 + 1.1s^5 + 1.1s^4 + 1.1s^3 + 0.08s^2 + 0.0024s} \qquad (24)$$

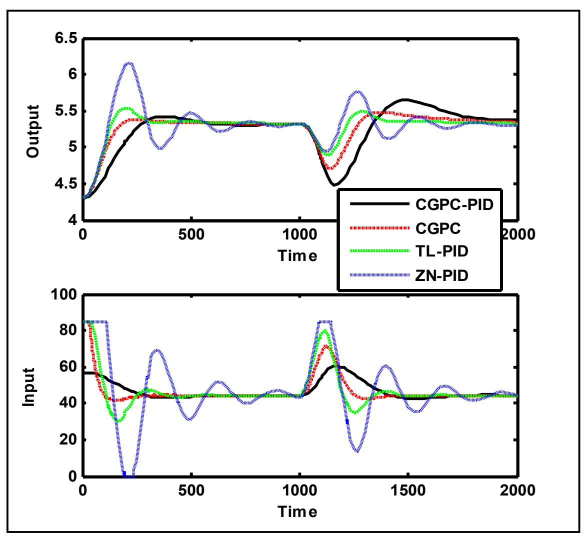

Fig. 7: Simulation Result for Example 4

Table 1: PID controller settings for Example 1: First Order Plus Delay System

Tuning methods					Set-Point		Disturbance	
	k_c	k_I	k_D	Ms	IAE	TV	IAE	TV
CGPC-PID	2.38	2.22	0.073	1.22	0.5015	2.5573	0.4345	1.08
CGPC	-	-	-	1.60	0.45	4.0746	0.4357	1.00
DS-PID(τ_c=2.8)	3.0856	4.2338	0.2565	1.14	0.5806	0.261	0.2362	1.30
ZN-PID	4.1526	9.051	0.4763	1.25	0.6539	12.08	0.1307	2.35

Table 2: PID controller settings for Example 2: Second Order Plus Delay System

Tuning methods					Set-point		Disturbance	
	k_c	k_I	k_D	Ms	IAE	TV	IAE	TV
CGPC-PID	3.76	0.25	6.35	2.08	9.86	7.78	3.93	1.75
CGPC	-	-	-	1.32	11.88	12.00	4.03	1.39
DS-PID(τ_c =0.5)	5	0.333	16.65	1.9	9.51	11.16	3.04	2.63
ZN-PID	4.72	0.8096	6.8912	2.365	10.15	16.74	1.75	2.98

Table 3: PID controller settings for Example 3: Non-Minimum Phase System

Tuning methods					Set-Point		Disturbance	
	k_c	k_I	k_D	Ms	IAE	TV	IAE	TV
CGPC-PID	0.879	0.0524	0.637	0.7	19.88	2.1902	23.34	1.8611
CGPC	-	-	-	1.95	19.04	2.7671	23.43	1.9199
DS-PID(τ_c=5)	0.947	0.0526	2.3684	0.125	19.60	3.2630	23.29	3.0127
ZN-PID	1.2	0.0988	3.6431	0.173	21.34	9.5058	17.44	6.0517

Table 4: PID controller settings for Example 4: Laboratory Scale Three-Tank-System

Tuning methods					Set-Point		Disturbance	
	k_c	k_I	k_D	Ms	IAE	TV	IAE	TV
CGPC-PID	16.4	0.03712	153.5	3.2	168.4	16.26	255.9	36.33
CGPC	-	-	-	1.45	114.6	101.68	146.5	72.07
ZN-PID	80.65	0.7676	2118.70	2.3	231.1	245.74	115.6	225.5
TL-PID	61.1	0.1322	2038.3	2.325	128.2	87.01	89.8	100.9

CGPC-PID controller was then obtained from (21) using equations (9) – (12).

The controller parameters and the performance metrics computed for CGPC law, CGPD-PID, TL-PID and ZN-PID are summarised in Table 4. CGPC law is found to have a lower value of Ms than others.

Figure 5 shows the simulation results obtained as a result of implementing the controllers on the nonlinear SIMULINK model of the system. Step changes in the set point at t=0 and in the disturbance at t=1000 seconds simulated as a leak of size 30 cm^3/sec in tank 1 and sustained for 100 seconds, were introduced. CGPC-PID approximated well the CGPC control law and provide a faster response than ZN-PID which was quite oscillatory.

CONCLUSION

We have presented in this work an analytical method of designing CGPC-based PID controllers. The comparison made in simulation between the original CGPC law and the approximated PID controller showed that Maclaurin series give a good approximation of the CGPC law. The simulation results obtained on application of the proposed design method on the four selected examples and comparing with DC-PID, T-L PID and Z-N PID controllers showed the effectiveness of the method.

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